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$2d$ stringy black holes and varying constants

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Abstract: Motivated by the recent interest on models with varying constants and whether black hole physics can constrain such theories, two-dimensional charged stringy black holes are considered. We exploit the role of two-dimensional stringy black holes as toy models for exploring paradoxes which may lead to constrains on a theory. A two-dimensional charged stringy black hole is investigated in two different settings. Firstly, the two-dimensional black hole is treated as an isolated object and secondly, it is contained in a thermal environment. In both cases, it is shown that the temperature and the entropy of the two-dimensional charged stringy black hole are decreased when its electric charge is increased in time. By piecing together our results and previous ones, we conclude that in the context of black hole thermodynamics one cannot derive any model independent constraints for the varying constants. Therefore, it seems that there aren't any varying constant theories that are out of favor with black hole thermodynamics.

Keywords: Black Holes in String Theory, 2D Gravity, Black Holes, Cosmology of Theories beyond the SM.

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Despite the fact that the idea of varying constants is more than forty years old [1]–[7] there has not been an experimental evidence as yet. The theoretical investigations that have been performed on this idea, can be roughly classified in two groups: (a) works in the framework of four-dimensional spacetimes where the fundamental constants were varying in space and/or in time, (b) and those works in higher dimensional spacetimes where the four-dimensional effective constants depend on any temporal or spatial variation of the structure or the size of extra-dimensions.[1] Regarding the former case, representative works are those of Bekenstein who treated the variability of the fine structure constant through a spacetime varying electron charge [9], and Moffat [10, 11] who in order to solve some cosmological problems introduced a time varying speed of light. Regarding now to the latter case and specifically within the context of string theory, the presence of the massless dilaton field which determines the string coupling constant $g_s = e^{\phi/2}$ and thus is the link between gravitation and matter interactions, led to violation of the universality of free fall and a time variation of the fine structure constant [2]. Damour and Polyakov in an attempt to reconcile massless dilaton field with the experimental tests of the equivalence principle presented a decoupling mechanism [13, 14].

Recent astronomical observations revived again the possibility of varying constants. Actually, there were hints that the fine structure constant $\alpha = e^2/\hbar c$ is increasing in time [15]–[17]. Precisely, by developing a new more sensitive (compared to the older "alkali-doublet") method called "many-multiplet", there has been a statistical evidence for a smaller α with $\Delta\alpha/\alpha = (-0.72 \pm 0.18) \times 10^{-5}$ for $z \approx 0.5 - 3.5$. Additionally, there were hints for a time variation of the electron to proton mass ratio $\mu = m_e/m_p$ [18, 19]. In particular, by measuring the H_2 wavelengths in the high-resolution of two quasars with damped Lyman-α systems at $z = 2.3377$ and $z = 3.0249$, they detected a time variation of μ with $\Delta\mu/\mu = (-5.7 \pm 3.18) \times 10^{-5}$ to be the most conservative result. The exact expression relating the fine structure constant α with the electron to proton mass ratio μ is still lacking but within the above-mentioned context presented by Damour and Polyakov the masses of electron and proton, and the fine structure constant depend on the massless dilaton field and hence are related. It should be noted that both measurements are of equivalent importance since they are non-zero detections.

The aforesaid astronomical results gave an impulse to new theoretical works on varying constants.[3] On one hand, expanding the previous works of Bekenstein and Moffat, Albrecht and Magueijo [21] presented as an alternative to Standard Big Bang model of the Universe in order to solve cosmological puzzles, a model of temporal varying speed of light (see

[1] The work presented here can be viewed within this framework. In particular, we treat two-dimensional black holes which are derived from a string theory heterotically compactified to two dimensions. Therefore, one can expect the time dependence of the two-dimensional constants to be a result of this heterotic compactification. Generally speaking, one cannot simply "write in" variations of constants since it is possible when a constant varies the black hole solution no longer exists [8].

[2] Recently, Bekenstein developed a mechanism which prevents equivalence principle violations due to variations of the fine structure constant α, to be measurable [12]. This compensating mechanism was introduced in the general framework discussed in [9].

[3] An excellent review concerning the fundamental constants and their variability is provided by Uzan [20].

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also [22]–[27]). On the other hand, Damour, Piazza and Venezianno obtained to extend the model of Damour and Polyakov [13, 14] in a way that the coupling functions which depend on the massless dilaton field, have a smooth finite limit for the infinitely large values of the bare string coupling [28, 29].

In an attempt to test theories of varying-c or varying-e, Davies, Davis and Lineweaver [30] considered as testing ground the black hole thermodynamics. They argued that the entropy of a four-dimensional Reissner-Nordström black hole solution of Einstein's theory of gravity decreases if the electric charge Q of this black hole increases while the Newton's constant G and c are kept constant. On the contrary, an increase of c will not lead to a violation of the second law of black hole thermodynamics.[4] Therefore, they concluded that theories in which the electric charge e varies in time, are disfavored since they violate the second law of thermodynamics. Immediately afterwards, Carlip and Vaidya [33] showed that when one considers the full thermal environment of the four-dimensional Reissner-Nordström black hole then no such conclusion is extracted. Fairbairn and Tytgat [34] also proved for the four-dimensional electrically charged black hole solution of string theory (well-known as GHS black hole) that its entropy remains constant with respect to adiabatic, i.e. slow, variations of the fine structure constant α, irrespectively of whether the change is due to an increase of e or an decrease of c.

In this paper, we consider the two-dimensional charged stringy black hole of McGuigan, Nappi and Yost [35]. Our motivation for this choice is the fact that the two-dimensional stringy black holes provide the simplified framework in which one can explore paradoxes such as the Hawking effect, that may lead to constrains on the theory [36]. Our starting point will be the two-dimensional effective action realized in heterotic string theory [37]

$$S = \int d^2x\sqrt{-g}e^{-2\phi}\left[R + 4(\nabla\phi)^2 + 4\lambda^2 - \frac{1}{4}F^2 + \nabla^a\left(4e^{-2\phi}\nabla_a\phi\right)\right], \tag{1}$$

where g is the determinant of the two-dimensional metric $g_{\mu\nu}(x)$, ϕ is the dilaton, λ^2 is the cosmological constant and $F_{\mu\nu}$ is the Maxwell stress tensor.

The line element of the two-dimensional charged stringy black hole solution derived by McGuigan, Nappi and Yost from action (1) is given, in the "Schwarzschild" gauge [38, 39], as

$$ds^2 = -g(r)dt^2 + g^{-1}(r)dr^2\,, \tag{2}$$

where

$$g(r) = 1 - 2me^{-2\lambda r} + q^2e^{-4\lambda r} \tag{3}$$

and the dilaton field is given as[5]

$$\phi(r) = \phi_0 - \lambda r \tag{4}$$

[4]The compatibility of varying c cosmologies with the (generalized) second law of thermodynamics was investigated in [31, 32].

[5]Apart from viewing equation (4) as a solution of action (1) for the dilaton field, one can also make two more comments for the linear dependence of dilaton field on the spatial coordinate. Firstly, the "linear" dilaton field is a common feature for lower-dimensional string theories [40]. For instance, in the case of CGHS black holes for zero mass ($M = 0$), one gets the "linear" dilaton vacuum which in the presence of any matter is perturbed and hence a formation of a black hole takes place [41, 42]. Secondly, one can treat

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with $0 < t < +\infty$, $r_+ < r < +\infty$, r_+ being the future event horizon of the black hole.

The constants m and q are related to the (ADM) mass M and electric charge Q_{el} of the two-dimensional charged stringy black hole, respectively, as

$$M = 4\lambda m e^{-2\phi_0} \tag{5}$$
$$Q_{el} = 2\sqrt{2}\lambda q e^{-2\phi_0} \tag{6}$$

where these values have been evaluated at infinity, i.e. $r \to \infty$.

It can be easily seen from action (1) that the fine structure constant is given as

$$\alpha = e^{2\phi_0}\,, \tag{7}$$

where ϕ_0 is the asymptotic value of the dilaton field.

At this point it should be pointed out that the fine structure constant as defined in four dimensions, i.e.

$$\alpha = \frac{e^2}{\hbar c}\,, \tag{8}$$

is no more dimensionless in the two dimensional spacetimes.[6] In particular, for the two-dimensional black hole backgrounds under consideration, since the metric function, the dilaton field and the action are considered as dimensionless one gets

$$[\lambda] = L^{-1}T^0\,,$$
$$[M] = L^{-1}T^0\,,$$
$$[Q_{el}] = L^{-1}T^0\,,$$

where [] denotes the dimension and L, T denote the length and time, respectively. It is noteworthy that in two dimensions the Newton's constant G_N is dimensionless and thus it is not possible to define even the "Planck" length.

The velocity of speed of light has dimensions

$$[c] = LT^{-1}$$

while the Planck's constant has dimensions

$$[\hbar] = LT^{-1}\,.$$

Thus, we define the two-dimensional fine structure constant[7] in analogy to four dimensions, by using the length scale $1/\lambda$, as follows

$$\alpha = \frac{\hbar\, e^2}{\lambda^2\, c}\,. \tag{9}$$

the dilaton field as the "radial" coordinate of the two-dimensional spacetime since it is linearly related to the spatial coordinate. For instance, the two-dimensional entropy can be written in terms of the value of the dilaton on the black hole horizon instead of the value of the radial coordinate [43].

[6]This is probably due to the super-renormalizability of QED in two dimensions [44].

[7]At this point, it should be stressed that although the two-dimensional fine structure constant introduced here is a link between gravitation and the electromagnetic properties of the two-dimensional black hole, it is not a priori related to atomic physics as the four-dimensional one. Thus, there is no obvious reason to relate the two-dimensional fine structure constant with the one we measure in the astrophysical systems.

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Pursuing a parametrization analogous to the four-dimensional case the metric function (3) factorizes as

$$g(r) = \left(1 - \rho_- e^{-2\lambda r}\right)\left(1 - \rho_+ e^{-2\lambda r}\right), \tag{10}$$

where

$$\rho_\pm = m \pm \sqrt{m^2 - q^2}\,. \tag{11}$$

It is easily seen that the outer event horizon H^+ is placed at the point $r_+ = \frac{1}{2\lambda} ln\rho_+$, while the "inner" horizon H^- is at the point $r_- = \frac{1}{2\lambda} ln\rho_-$.

The temperature of the two-dimensional charged stringy black hole can easily be derived by implementing its definition [45]

$$T_H = \frac{\kappa}{2\pi} \qquad \text{and} \qquad \kappa = \frac{1}{2}\left.\frac{\partial g(r)}{\partial r}\right|_{r=r_+} \tag{12}$$

which yields the following expression for the Hawking temperature [39]

$$T_H = \frac{\lambda\sqrt{m^2 - q^2}}{\pi\left(m + \sqrt{m^2 - q^2}\right)}. \tag{13}$$

From thermodynamics [37], we can also obtain the entropy of the two-dimensional charged stringy black hole

$$\begin{aligned} S &= 4\pi e^{-2\phi_0}\left(m + \sqrt{m^2 - q^2}\right) \\ &= 4\pi m e^{-2\phi_0}\left(1 + \sqrt{1-\gamma}\right), \end{aligned} \tag{14}$$

where

$$\gamma = \left(\frac{q}{m}\right)^2 \tag{15}$$

is a dimensionless parameter which in terms of the mass M and the electric charge $Q_{\rm el}$ of the two-dimensional charged stringy black hole is given as

$$\gamma = 2\left(\frac{Q_{\rm el}}{M}\right)^2 \tag{16}$$

and the electric charge $Q_{\rm el}$ is quantized in units of the electric charge e, i.e. $Q_{\rm el} = ne$.[8] It is obvious that the temperature (13) of the two-dimensional charged stringy black hole can now be written in terms of the dimensionless parameter γ as

$$T_H = \frac{\lambda}{\pi}\left(1 + \frac{1}{\sqrt{1-\gamma}}\right)^{-1}. \tag{17}$$

It should be pointed out that the entropy (14) and the temperature (17) have both been evaluated on the outer horizon, i.e. $r = r_+$, of the two-dimensional charged stringy black

[8] Up to now, a full theory of quantum gravity does not exist thus black hole mass is not treated as quantized, i.e. $M = \sqrt{r} M_{pl}$ (r is discrete here), and furthermore the mass quantization may lead to a discrete evolution of the fine structure constant α [46].

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hole. Since we are going to consider the increase of the fine structure α solely due to an increase in the electric charge e all other quantities are kept constant. It is worthy to note that the reason for introducing the dimensionless parameter γ in the expressions for the temperature and the entropy of the two-dimensional charged stringy black hole is the fact that one can only consider variation of dimensionless constants.[9] Thus, by varying e we mean here the variation of dimensionless quantities that depend on e.

It is obvious from expression (15) that an increase of the electric charge e implies an increase in γ. Thus, concerning the temperature of the two-dimensional stringy black hole, it is easily derived from expression (17) that an increase in γ causes the temperature to decrease. Therefore, as the electric charge e increases the two-dimensional stringy black holes becomes cooler. It can also be checked from expression (14) that an increase in γ will lead the entropy of the two-dimensional charged stringy black hole to become smaller. Therefore, an increase in the electric charge e in the specific black hole background is at the risk of violating the second law of black hole thermodynamics. At first sight, this seems to be catastrophic. Consequently, one is considering to rule out varying e theories in order to avoid the aforesaid violation. However, since we are concerned with physical observations, one should not be interested in studying an isolated black hole rather than a black hole in its thermal environment. Therefore, we now consider the two-dimensional charged stringy black hole contained in a "box" and the physical measurements will be made on the boundary of this black hole spacetime, i.e. on the "wall" of the "box" which is located at a finite distance of the radial coordinate. It is convenient to follow the terminology of Gibbons and Perry [49] and the action is now given as

$$S = \int_{\mathcal{M}} d^2x\sqrt{-g}e^{\phi}\left[R + (\nabla\phi)^2 + 4\lambda^2 - \frac{1}{4}F^2\right] + 2\int_{\partial\mathcal{M}} e^{\phi}Kd\Sigma\,, \tag{18}$$

where K is the trace of the second fundamental form of the boundary, i.e. $\partial\mathcal{M}$, of the two-dimensional spacetime $\mathcal{M}$ and $d\Sigma$ is the volume on the boundary. It is evident that the fine structure α takes the form

$$\alpha = e^{-\phi_w}\,, \tag{19}$$

where ϕ_w is the value of the dilaton field on the boundary, i.e. the "wall" of the "box". The line element is given, in the unitary gauge [39], by

$$ds^2 = -\frac{\left(m^2 - q^2\right)\sinh^2(2\lambda y)}{\left(m + \sqrt{m^2 - q^2}\cosh^2(2\lambda y)\right)^2}dt^2 + dy^2\,, \tag{20}$$

where the unitary variable is given by

$$y = \frac{1}{\lambda}\ln\left[\sqrt{\frac{1}{\mu}\left(e^{2\lambda(r-r_+)} - 1\right)} + \sqrt{\frac{1}{\mu}\left(e^{2\lambda(r-r_+)} - 1\right) + 1}\right] \tag{21}$$

and $0 < y < +\infty$, while $\mu = 1 - \rho_-/\rho_+$.

[9] Lately, there was a debate on whether is physically meaningful to consider time variation of dimensional constants [47, 48].

The dilaton field evaluated on the wall, i.e. $y = y_w$, will be given as

$$\phi_w = \phi_0 + \ln\left[\frac{1}{2}\left(\frac{1}{\sqrt{1-\gamma}} + \cosh(2\lambda y_w)\right)\right] \tag{22}$$

while the dilaton charge is

$$D = \frac{1}{2}e^{\phi_0}\left(\frac{1}{\sqrt{1-\gamma}} + \cosh\left(2\lambda y_w\right)\right). \tag{23}$$

The electric charge $Q_{\rm el}$ and the (ADM) mass M of the two-dimensional charged stringy black hole evaluated inside the "box" are given, respectively, as

$$M = \frac{2\lambda}{\sqrt{1-\gamma}}e^{\phi_0} \tag{24}$$

$$Q_{\rm el} = \frac{\sqrt{2}\lambda}{m}\frac{\sinh\left(2\lambda y_w\right)}{\left[1 + (1-\gamma)\cosh\left(2\lambda y_w\right)\right]}e^{\phi_0}. \tag{25}$$

The local temperature of a self-gravitating system in thermal equilibrium is given by Tolman's law as

$$T_{local} = \frac{T_H}{\sqrt{-g_{tt}}}, \tag{26}$$

therefore the temperature of the two-dimensional charged stringy black hole evaluated on the "wall" of the "box" is

$$T_w = T_H\left(\frac{1 + \sqrt{1-\gamma}\cosh\left(2\lambda y_w\right)}{\sqrt{1-\gamma}\sinh\left(2\lambda y_w\right)}\right). \tag{27}$$

The entropy of the two-dimensional charged stringy black hole is given by

$$S = -\left(\frac{\partial F}{\partial y_w}\right)_{\lambda,D,Q_{\rm el}}\left(\frac{\partial T_w}{\partial y_w}\right)^{-1}_{\lambda,D,Q_{\rm el}}, \tag{28}$$

where the free energy $F = F(\lambda, D, T_w, y_w)$ is

$$F = -4\lambda D\coth\left(2\lambda y_w\right) \tag{29}$$

and thus the explicit expression for entropy is

$$S = 4\pi e^{\phi_w}\frac{\left(1 + \frac{1}{\sqrt{1-\gamma}}\right)}{\left(1 + \frac{\cosh(2\lambda y_w)}{\sqrt{1-\gamma}}\right)}, \tag{30}$$

where equations (17), (22), (23) and (27) have been substituted in (30). It is clear that even if someone takes into consideration the fact that the entropy is affected by a change in the boundary value ϕ_w of the dilaton field and thus affected by the variation of α as seen from equation (19), the entropy is getting smaller with respect to an increase in γ, i.e. an increase in the electric charge e. Therefore, although we have concerned the two-dimensional charged stringy black hole in a thermal environment, i.e. in the "box", an increase in the electric charge causes the entropy of the above mentioned gravitational background to be smaller. The violation of the second law of thermodynamics seems to be inevitable.

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In summary, we have viewed a two-dimensional stringy black hole as an isolated object and also as contained in a thermal environment. In both cases, it was shown that an increase of the electric charge e leads to an expected decrease of the temperature and to an unexpected decrease of the entropy of the two-dimensional charged stringy black hole. The latter result implies that varying e theories run the risk of violating the second law of thermodynamics.

Therefore, we believe that our results are a signal of the specific model, c.f. [50, 46, 33, 34], meaning that results derived in the context of black hole thermodynamics will always be model-dependent and will not lead to any constrains on varying constant theories. Of course, one could claim that the results derived here are a support for the arguments of Davies et al. [30] that black holes are able to discriminate between the varying e and varying c theories. But in this case, one has a priori to believe that experiments measure dimensional quantities which is completely erroneous [47].

Finally, a couple of points are in order. Firstly, the analysis presented here has taken for granted the increase of the fine structure α in time although a confirmation of this variation is still lacking in astrophysical observations. Secondly, we have assumed that the variation of the fine structure α is due to an increase in the electric charge e.[10] There is also the possibility that the variation of α could be due to a variation in the speed of light c or due to a variation of the Planck's constant $\hbar$ or even due to a simultaneous variation of some of the aforesaid constants [27].

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